

88047401

MATHEMATICS
HIGHER LEVEL
PAPER 1

Wednesday 3 November 2004 (afternoon)

2 hours

School code

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Candidate code

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INSTRUCTIONS TO CANDIDATES

- Write your school code and candidate code in the boxes above.
- Do not open this examination paper until instructed to do so.
- Answer all the questions in the spaces provided.
- Unless otherwise stated in the question, all numerical answers must be given exactly or to three significant figures.
- Indicate the make and model of your calculator in the appropriate box on your cover sheet.

Maximum marks will be given for correct answers. Where an answer is wrong, some marks may be given for correct method, provided this is shown by written working. Working may be continued below the box, if necessary. Solutions found from a graphic display calculator should be supported by suitable working, e.g. if graphs are used to find a solution, you should sketch these as part of your answer.

1. Consider $f(x) = x^3 - 2x^2 - 5x + k$. Find the value of k if $(x + 2)$ is a factor of $f(x)$.

Working:

Answer:

2. Given that the matrix $A = \begin{pmatrix} 1 & -1 & 2 \\ 2 & p & 3 \\ 1 & -2 & 5 \end{pmatrix}$ is singular, find the value of p .

Working:

Answer:

3. The sum of the first n terms of a series is given by

$$S_n = 2n^2 - n, \text{ where } n \in \mathbb{Z}^+.$$

- (a) Find the first three terms of the series.
- (b) Find an expression for the n^{th} term of the series, giving your answer in terms of n .

Working:

Answers:

(a) _____

(b) _____

4. Given that $(a + i)(2 - bi) = 7 - i$, find the value of a and of b , where $a, b \in \mathbb{Z}$.

Working:

Answer:

5. If $y = \ln(2x - 1)$, find $\frac{d^2y}{dx^2}$.

Working:

Answer:

6. A fair six-sided die, with sides numbered 1, 1, 2, 3, 4, 5 is thrown. Find the mean and variance of the score.

Working:

Answer:

7. (a) Find the largest set S of values of x such that the function $f(x) = \frac{1}{\sqrt{3-x^2}}$ takes real values.
- (b) Find the range of the function f defined on the domain S .

Working:

Answers:

- (a) _____
- (b) _____

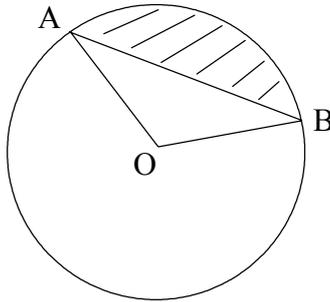
8. (a) Find the expansion of $(2+x)^5$, giving your answer in ascending powers of x .
- (b) By letting $x = 0.01$ or otherwise, find the **exact** value of 2.01^5 .

Working:

Answers:

- (a) _____
- (b) _____

9. The diagram below shows a circle centre O and radius OA = 5 cm. The angle AOB = 135°.



Find the area of the shaded region.

Working:

Answer:

10. Consider the equation $e^{-x} = \cos 2x$, for $0 \leq x \leq 2\pi$.

(a) How many solutions are there to this equation?

(b) Find the solution closest to 2π , giving your answer to **four** decimal places.

Working:

Answers:

(a) _____

(b) _____

11. Consider the four points A(1, 4, -1), B(2, 5, -2), C(5, 6, 3) and D(8, 8, 4). Find the point of intersection of the lines (AB) and (CD).

Working:

Answer:

12. A continuous random variable X has probability density function given by

$$f(x) = k(2x - x^2), \quad \text{for } 0 \leq x \leq 2$$
$$f(x) = 0, \quad \text{elsewhere.}$$

- (a) Find the value of k .
- (b) Find $P(0.25 \leq x \leq 0.5)$.

Working:

Answers:

- (a) _____
- (b) _____

13. Given that $z \in \mathbb{C}$, solve the equation $z^3 - 8i = 0$, giving your answers in the form $z = r(\cos \theta + i \sin \theta)$.

Working:

Answer:

14. Find the total area of the two regions enclosed by the curve $y = x^3 - 3x^2 - 9x + 27$ and the line $y = x + 3$.

Working:

Answer:

15. Find the range of values of m such that for all x

$$m(x+1) \leq x^2.$$

Working:

Answer:

16. Find the equation of the normal to the curve $x^3 + y^3 - 9xy = 0$ at the point (2, 4).

Working:

Answer:

17. Using the substitution $2x = \sin \theta$, or otherwise, find $\int (\sqrt{1-4x^2}) dx$.

Working:

Answer:

18. A closed cylindrical can has a volume of 500 cm^3 . The height of the can is h cm and the radius of the base is r cm.
- (a) Find an expression for the total surface area A of the can, in terms of r .
- (b) Given that there is a minimum value of A for $r > 0$, find this value of r .

Working:

Answers:

(a) _____

(b) _____

19. (a) Find the cartesian equation of the plane that contains the origin O and the two points A(1, 1, 1) and B(2, -1, 3).
- (b) Find the distance from the point C(10, 5, 1) to the plane OAB.

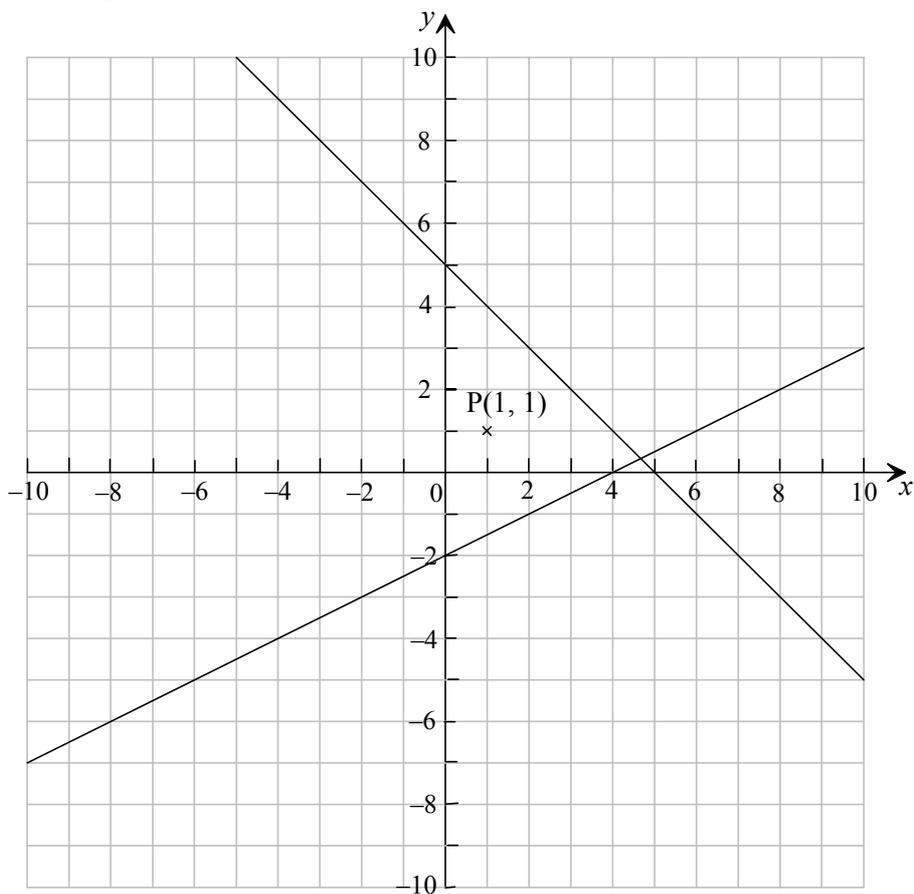
Working:

Answers:

(a) _____

(b) _____

20. The following diagram shows the lines $x - 2y - 4 = 0$, $x + y = 5$ and the point $P(1, 1)$. A line is drawn from P to intersect with $x - 2y - 4 = 0$ at Q , and with $x + y = 5$ at R , so that P is the midpoint of $[QR]$.



Find the **exact** coordinates of Q and of R .

Working:

Answer: